Quantitative Finance

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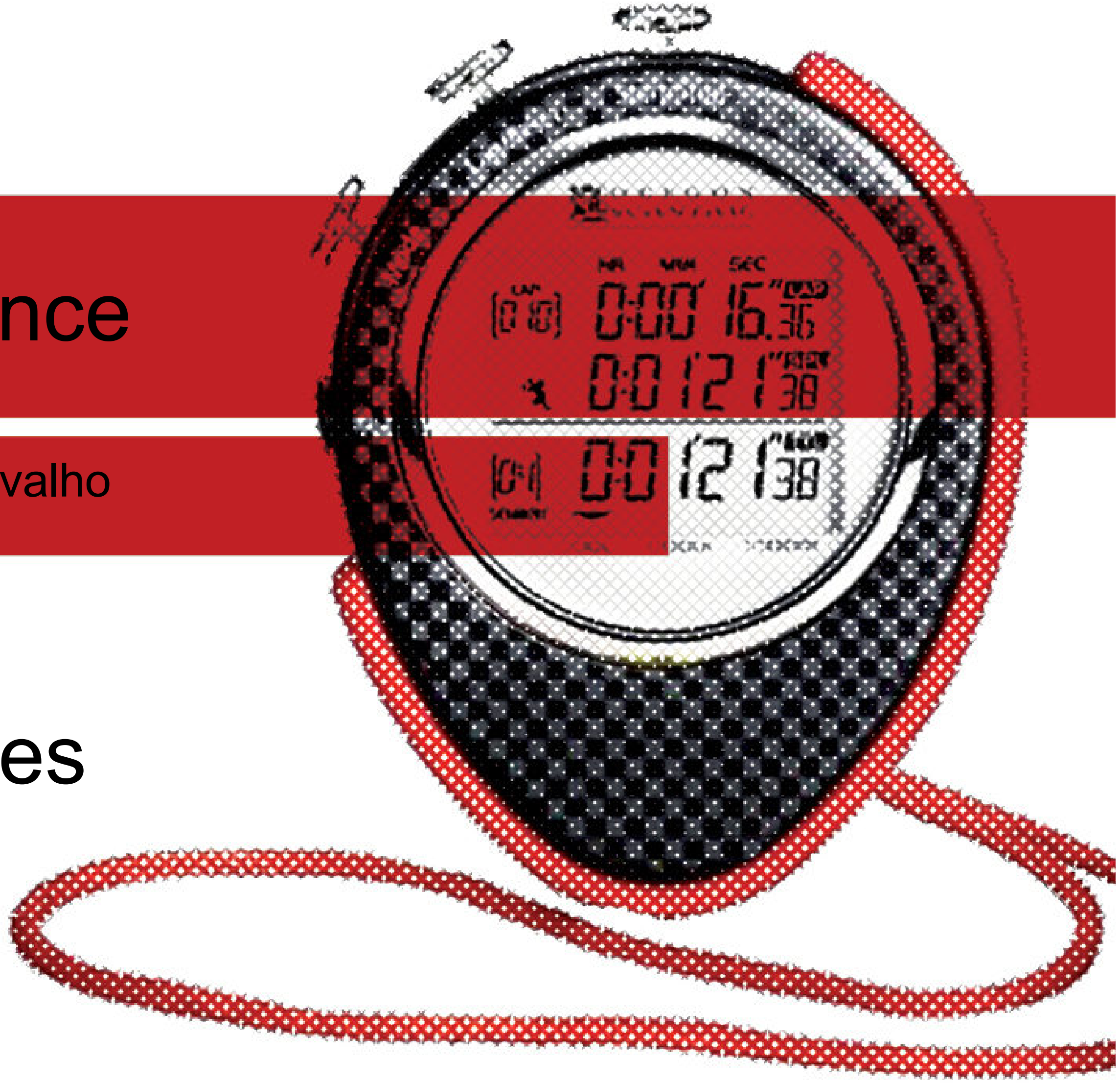
Other Annuities Certain



ISEG 100 1911-2011 ANOS

100 ANOS A PENSAR NO FUTURO

Deferred annuities

An Annuity with the present value located two or more periods before the first payment is called a deferred annuity.

PV R R R R R R

0 1 2 3 4 5 6.... n

$PV = R \times a_{n|i} \, (1+i)^{-m}$; where **m** is the number of intervals of deferment, in this example m=~~2.~~ =1

$FV = PV \, (1+i)^{n}$

Comparing ordinary annuities and deferred annuities

Ordinary annuity (i=0.01)

0 1 2 3 4 5 6

|____|____|____|____|____|____|_______|_

500 500 500 500 500 500

$PV_0 = 500 \times a_{6|0.01} = 2897.74$

$FV_{\mathbf{6}} = 500 \times S_{6|0.01} = 3076.01$

Deferred annuity ((i=0.01; m=2)

0 1 2 3 4 5 6 7 8

|_____|_____|_____|_____|_____|_____|_____|_____|

500 500 500 500 500 500

$PV_0 = 500 \times a_{6|0.01} \times (1+0.01)^{-2} = 2840.64$

$FV_{\mathbf{8}} = 500 \times S_{6|0.01} = 3076.01$

Forborne annuities

An Annuity with the future value located p periods after the last payment.

	R	R	R	R	R	R	FV
0	1	2	3	4	5	6....	p

$FV = R \times S_{n|i} (1+i)^p$; where p is the number of intervals between the last payment and FV.

PERPETUITIES

A perpetuity is an annuity with infinite term in which the periodic payments are the interest earned during the previousinterest period. A perpetuity has only a present value.

Ordinary perpetuity

$a_{\infty|i} = \lim_{n \to \infty} (a_{n|i}) = 1/i$

$PV = R/i$

Perpetuity due

$PV = R/i \times (1+i) = R + R/i$

Deferred perpetuity (m periods)

$PV = R/i \times (1+i)^{-m}$

PRESENT VALUE – GROWING ARITHMETIC PROGRESSION

Arithmetic progression: This is a sequence which has a starting number C and successive numbers are obtained by adding a number h (called the common difference or constant):

$$c, c + h, c + 2h, \cdot\cdot\cdot, c + (n - 1)h$$

Ex: 1000, 1500, 2000, 2500 - First term is 1000 and the constant is 500.

Present value when first term is C and the constant is h:

PV = (C-h) an|i + h (Ia)n|i

Where (Ia)n|i = [än|i – n (1+i)-n] / i

PRESENT VALUE – DECREASING ARITHMETIC PROGRESSION

Ex: 2500, 2400, 2300, 2200 - Last term is 2200 and the constant is 100.

Present value when last term is D and the constant is h:

$$PV = (D-h)\, a_{n|i} + h\, (Da)_{n|i}$$

Where $(Da)_{n|i} = (n - a_{n|i}) / i$

PRESENT VALUE – GEOMETRIC PROGRESSION

Geometric progression: The geometric progression has a starting number C and successive terms are obtained by multiplying by a common ratio r called the factor:

$$C,\ Cr,\ Cr^2,\ \cdots,\ Cr^{(n-1)}$$

Ex: 100, 200, 400, 800 - First term is 100 and the factor is 2

Factor > 1 growing progression.

Factor < 1 decreasing progression

Present value when first term is C and the factor is r:

- $PV = C\ [\ 1\text{-}r^n\ (1+i)^{-n}]\ /\ (1+i\text{-}r)$

PV = C [1-rn (1+i)-n] / (1+i-r)